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Abstract

We consider issues concerning unemployment insurance under establishment dynamics. Extending the framework developed by Alvarez and Veracierto (1998), we investigate the welfare consequences of unemployment insurance for the aggregate economy and different groups of households. Although the aggregate implications of our quantitative results are similar to those of previous research, we show that unemployment insurance has different effects for agents with high and low asset holdings. In particular, increasing the replacement rate benefits agents with nearly zero assets and holders of very high levels of assets. Furthermore, we demonstrate that a different construction of the disutility of search effort has significant implications for unemployment insurance. If the search effort is inelastic over assets, the effect of the insurance declines compared with the case of flexible search.

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1 Introduction

We consider the risk-sharing effects of unemployment insurance (UI) under establishment dynamics. Although many papers, including Abdulkadiroğlu, Kuruşçu, and Şahin (2002), Wang and Williamson (2002), and Young (2005), investigate the optimal unemployment insurance under different setups, little is known about the welfare consequences of unemployment insurance when establishment dynamics are explicitly considered. As the setup significantly influences the demand for aggregate labor and capital, the welfare impact of the unemployment insurance must be carefully reconsidered under general equilibrium. In addition, little attention has been given to the point that the insurance has different effects on several levels of asset holdings. In this paper, we focus on the insurance effects for agents whose levels of asset holdings are heterogeneous in the general equilibrium. Moreover, we investigate the relationship between the endogenously determined wealth distribution and search effort.

First, we examine under which replacement rate (i.e., the benefit level relative to the wage) the social welfare is maximized if the unemployed agents can receive unemployment insurance for one quarter. For this purpose, we compare the steady-state results of the general equilibrium under several replacement rates. Second, we investigate the welfare consequences of those policy changes on different groups of agents. As the asset distribution is endogenously generated, we need to consider the partial and general equilibrium effects of the unemployment insurance. To separate those effects, we evaluate the agents' welfare at some fixed benchmark asset levels, and at endogenously determined several percentile points of the asset distribution. Finally, we consider how the elasticity of the search effort over the asset holdings affects the equilibrium outcome. If the effort function is constant over the assets, the unemployment insurance has little effect on average agents, but it has a large effect on agents with zero assets.

Following the research by Davis, Haltiwanger, and Schuh (1996), the main focus of labor market research has spread beyond macro statistics of the unemployment rate to encompass the rates of job creation and destruction (JCR/JDR), defined precisely in their book. Because employment positions are mainly determined by the decisions of firms, in considering *unemployment risks*, we should consider the firms' behavior explicitly, and include JCR/JDR in the analysis. However, most search models on unemployment insurance omit those features for simplicity, or for analytical reasons.[1] We investigate the welfare effects of unemployment insurance by using

[1] A few exceptions are Alvarez and Veracierto (1998, 2001) and Alonso-Borrego, Fernández-Villaverde, and Galdón-Sánchez (2005). Alvarez and Veracierto (2001) focus on the insurance effect of severance payments and explain high unemployment rates in several European countries. They demonstrate that the severance payments have a positive role as an alternative to unemployment insurance. In addition, in a working version of their paper, they investigate the effect of unemployment insurance. Alonso-Borrego et al. (2005) explain the high unemployment rate in the Spanish economy by including temporary jobs in their model.

the stochastic dynamic general equilibrium model developed by Alvarez and Veracierto (1998). Their model involves infinitely many firms and households, which include workers and searchers.

We employed this model for two reasons: (1) the unemployment risk is endogenously determined, and (2) firms' structures of production are explicitly modeled. Under the model, the unemployment problem occurs as result of heterogeneous firms' decisions on hiring and firing, which results in agents who are dismissed needing to search for a job. Therefore, instead of calibrating the unemployment risks directly, as done by Hansen and İmrohoroğlu (1992) and other papers, we can model the unemployment risks based upon the firms' decisions. As the model includes capital accumulation, we investigate the relationship between agents' levels of asset holdings and the welfare effect of unemployment insurance, measured by certainty equivalent consumption. Furthermore, the model implies that demand for both labor and capital has strong effects on the market equilibrium outcomes.

Our model differs in several respects from that of Alvarez and Veracierto (1998). In particular, we made two significant modifications, altering the timing of search after dismissal and bequests. In Alvarez and Veracierto (1998), households hold assets for precautionary motives and for their retirement. Thus, Alvarez and Veracierto (1998) include the life cycle motive in their model, and all accidental bequests after death are taken over by the household's descendants. As a consequence, almost all households have sufficient wealth to share unemployment risks. Although the life cycle motive is realistic, it may not be adequate to investigate the welfare consequences of unemployment insurance. Often, the unemployed do not hold sufficient wealth to hedge the risks. Therefore, an incremental increase of unemployment insurance may have a positive role. The search opportunity provided by the unemployment insurance makes the duration of the unemployment short. This implies that damage caused by the disutility of effort is not so high compared with the setup by Alvarez and Veracierto (1998) even if the unemployment rate is the same.

We show that social welfare is maximized at 0% of the replacement rate. Although this finding is fairly close to the findings of recent literature on unemployment insurance, the policy has different implications if we carefully analyze the insurance effects for different groups of agents. In particular, changes in the unemployment insurance policy improve welfare for very poor and wealthier agents through high marginal utility and general equilibrium effects. Therefore, even if the optimal replacement rate is zero for an *average* agent, unemployment insurance can improve welfare for some workers. Moreover, the attitude toward the search effort quantitatively changes the effect of the unemployment insurance. If the search effort is inelastic over the asset holdings, the effect of the unemployment insurance declines owing to a low demand for labor by the firms.

Recently, many papers have addressed the optimal unemployment insurance by investigating the impacts of the insurance quantitatively. Such research includes Hansen and İmrohoroğlu

(1992), Abdulkadiroğlu et al. (2002), Wang and Williamson (2002), and Young (2005).[2] Some of the papers focus on the pure effect of the unemployment insurance by using a model without savings. However, such research may disregard precautionary savings, the importance of which has recently been empirically underlined by Gourinchas and Parker (2002). In the recent literature, Young (2005) constructs a search model with capital accumulation and reveals that a zero replacement rate of unemployment insurance attains the highest expected utility, i.e., it is optimal for the stationary states.[3] Moreover, Young (2005) demonstrates that very few agents are disadvantaged on the transition path to a zero-unemployment insurance system. In our paper, we reconsider Young's results using a different model. Although the average agent's welfare is improved at the optimal replacement rate compared with the current system, the welfare gain is small. The level of the welfare gain is close to that found in the recent literature. For example, Wang and Williamson (2002) demonstrate that although the change from the current system to the optimal system can improve welfare, the welfare gain measured by consumption is only 0.09%. The result of Abdulkadiroğlu et al. (2002) is much larger, amounting to 2.01% when saving is prohibited and 0.184% when hidden saving is possible. To date, few attempts have been made at examining the relationship between job search and savings. One exception is Lentz and Tranæs (2005).

The paper is organized as follows. In Section 2, we construct our model. In Section 3, we select calibrated parameters carefully to match the US economy. In Section 4, we discuss numerical results under general equilibrium, and we extend the investigation for several levels of assets in Section 5. Finally, Section 6 concludes the paper.

2 The Model

We consider an infinite-horizon economy with a continuum of heterogeneous households and firms. Each firm faces productivity uncertainty and determines how many workers it hires and how much capital it rents. If a firm decides to dismiss workers who were employed in the last period, this subset of the workers becomes unemployed. Although the unemployed households need to search for a new job to obtain labor income, demand for new workers is endogenously determined by the firms. Therefore, compared with the usual search models, the unemployment rate is largely governed by the firms' behavior. Under this setup, we focus on a stationary equilibrium. Our setup is based upon Alvarez and Veracierto (1998, 2001) and Alonso-Borrego

[2]There are many theoretical research studies concerning optimal unemployment insurance. See, for example, Hopenhayn and Nicolini (1997), Kocherlakota (2003), and Werning (2002). Those papers consider only the case of transition from unemployment to employment, namely a situation where the employment state is absorbing. Fredriksson and Holmlund (2003) provide an excellent survey on this topic.

[3]Such a policy was previously advocated by Alvarez and Veracierto (1998) in a different setup.

et al. (2005).

2.1 A Firm's Behavior

There is a continuum of firms, and each firm faces idiosyncratic production risks, $s_t \in \mathcal{S} \subset \mathbb{R}_+$, which take discrete values. At period t, each firm is distinguished by a pair of states (n_{t-1}, s_t): how much the firm employed in the last period, $n_{t-1} \in \mathcal{N} \subset \mathbb{R}_+$, and the productivity level realized at the beginning of this period. After observing the realization of the productivity shocks, given n_{t-1}, the firm decides how many workers to employ (namely, hiring n_t) and renting capital, k_t. All firms have the same production function, $s_t k_t^{\theta} n_t^{\eta}$, which is assumed to involve decreasing return to scale, i.e., $\theta + \eta < 1$.[4] The parameter θ represents a share of capital and the parameter η is a share of labor.

Although firms can choose to dismiss some fraction of their workers, firing involves costs. In the US, Japan, or any other developed country, it is well known that firing costs are often high.[5] We assume that some fraction of workers, σn_{t-1} $(0 < \sigma < 1)$, retires in each period, and that they leave the economy. Hence, the firms can decide to reduce employment without cost if they are prepared to wait for a long time. Otherwise, if the firms want to decrease the employment level radically, such as might occur in the case where there is a large productivity decline, they must expend huge costs to dismiss the workers. For simplicity, we assume that the capital input for production is freely changeable without extra frictional cost; that is, the capital input is always optimal. The labor adjustment cost is given by:

$$h\left(n_t, n_{t-1}\right) = w_t \left(\tau^h I\left[n_t > (1-\sigma)n_{t-1}\right] - \tau^f I\left[n_t < (1-\sigma)n_{t-1}\right]\right)\left(n_t - n_{t-1}\right), \qquad (1)$$

where w_t describes a wage, and $I[\cdot]$ is an indicator function. If the statement in the parentheses $[\cdot]$ is true, then the value becomes one; otherwise it is zero. Multiplied parameters (τ^h, τ^f) are the job creation and destruction cost per worker measured by the wage. The costs are paid by consumption goods and are assumed to be discarded for simplicity. Following Caballero, Engel, and Haltiwanger's (1997) research on adjustment costs, we assume that the function form is linear.[6] For example, if a firm wants to dismiss some workers excluding the retiree σn_{t-1}, it costs $w\tau^f$ per worker. Note that the firms need not charge job destruction costs when the employers retire at period t. A simple interpretation of the adjustment cost is severance payments, which is well discussed by Alvarez and Veracierto (1998, 2001).

[4] Lucas (1978) considers an endogenous determination problem of firm size distribution when the number of the firms is a continuum measure. The parameter $(1 - \theta - \eta)$ is called the "span of control."

[5] Lazear (1990) investigates severance payments among several countries and concludes that such a payment reduces employment.

[6] Cooper, Haltiwanger, and Willis (2004) build several types of cost functions in their model and estimate the structural parameters of the model.

Taking into account the adjustment cost function of Equation (1), given an interest rate r_t and a wage, a firm's cash flow $\pi(n_{t-1}, s_t)$ is defined as follows:

$$\pi(n_{t-1}, s_t) = s_t k_t^{\theta} n_t^{\eta} - (r_t + \delta) k_t - w_t n_t - h(n_t, n_{t-1}),$$

where δ is depreciation. We assume that workers are identical and that they cannot observe the types of firms before working for the firm. Under this assumption, the firms have no incentive to offer different wages among workers when hiring. Thus, the wage is the same for all firms.

There are no insurance markets concerning productivity shocks, and firms cannot share the risks before realization of the shocks. Therefore, the firms' decisions and the stochastic productivity process govern workers' employment risks. We assume that there are no fixed costs for running the firms, and that productivity is positive for all states, i.e., $s_t > 0$. Thus, we exclude the entry and exit of firms in our model. Although this is an important feature when studying job creation and destruction, its omission does not result in loss of generality when the purpose is to investigate the welfare effects of unemployment insurance, as in our case.[7]

Each firm selects the level of current labor inputs in order to maximize the expected discounted profit, defined as follows:

$$\max_{\{n_t\}} E \sum_{t=0}^{\infty} \beta^t \pi(n_{t-1}, s_t), \text{ given } n_{-1},$$

where β is the discount factor of the employer of the firm. Under a recursive environment, we omit the time subscript of the model. Given the states (n, s), the Bellman equation of a firm's problem is defined as follows:

$$V(n, s) = \max_{n'} \{\pi(n, s) + \beta E V(n', s')\}, \tag{2}$$

where $V : \mathcal{N} \times \mathcal{S} \to \mathbb{R}$ is a value function. The pair of states (n', s') represents the next period's states, and E is an expectation operator. We assume that profits are consumed by the employer, not distributed to households. This assumption results in an increasing number of households with assets close to zero.

A firm's decision depends solely on the states, and we denote the policy function as $n' = g(n, s)$. We assume the idiosyncratic productivity shocks follow a finite Markov chain, $Q(s_{t+1}, s_t)$, and that there exists a stationary distribution of the stochastic process. From the transition matrix and the policy function, we obtain the size distribution of the firms.

[7] Hopenhayn and Rogerson (1993) consider the entry and exit of firms using a general equilibrium model in an environment where firms' manager must pay fixed costs for each period to run a firm. On the other hand, Alvarez and Veracierto (1998) consider the case when productivity is zero, which implies the death of the firm.

2.1.1 Size Distribution of the Firms

By normalizing the total mass of the firms to be one, the transition probability $Q(s_{t+1}, s_t)$ represents the fraction of firms which transit from productivity s_t to s_{t+1}, by the law of large numbers. We denote the number of firms with type (n_{t-1}, s_t) as $\phi(n_{t-1}, s_t)$. The law of motion for the size distribution is given as follows.

$$\mathcal{Q}\left[(B, s_{t+1}), (n_{t-1}, s_t)\right] = \begin{cases} Q(s_{t+1}, s_t), & \text{if } g(n_{t-1}, s_t) \in B \subseteq \mathcal{N}, \\ 0, & \text{otherwise} \end{cases}$$

$$\phi'(B, s') = \mathcal{T}\phi = \int \mathcal{Q}\left[(B, s'), (n, s)\right] \phi(dn, ds)$$

We focus on a time-invariant distribution of the size distribution, i.e., $\phi' = \mathcal{T}\phi$. The existence of the stationary distribution is guaranteed by Theorem 11.12 in Stokey, Lucas, and Prescott (1989).

In order to construct the employment dynamics of the agents, we define a distribution of the firms of expanding their size (namely, increasing n_{t+1} from $(1-\sigma)n_t$) as $\Phi^+(n_t, s_{t+1})$. By defining the total number of the newly employed as $n^+ = \int \Phi^+(n_t, s_{t+1})(dn, ds)$, we define $\phi^+(n_t, s_{t+1}) \overset{\text{def}}{=} \frac{\Phi^+(n_t, s_{t+1})}{n^+}$.

Because the capital market is assumed to be frictionless, the capital demand of a firm is determined by the condition that marginal productivity is equal to the interest rate; that is, $k_t = k^d(n_{t-1}, s_t) = \left[\frac{\theta s_t n_t^\eta}{r_t + \delta}\right]^{\frac{1}{1-\theta}}$.

Given an interest rate and wage, the aggregate demands for labor N_t^D and capital K_t^D are defined as an integration of the firm's input decision using the size distribution:

$$N_t^D = \int_{\mathcal{N} \times \mathcal{S}} g(n_{t-1}, s_t)\, \phi(n_{t-1}, s_t)(dn, ds),$$

$$K_t^D = \int_{\mathcal{N} \times \mathcal{S}} k^d(n_{t-1}, s_t)\, \phi(n_{t-1}, s_t)(dn, ds).$$

At the stationary state on which we focus, the aggregate variables do not change over time.

2.2 Households' Behavior

We construct households' behavior to be consistent with firms' decisions, as explained above. The households are categorized into two types: workers and searchers. As we assumed above, some agents leave the labor market at the end of a period when they die, and they are replaced by their descendants. New entrants start economic activities with zero assets. Whether workers retain an employment relationship depends upon the decisions of the firms by which they are hired. As workers are identical in terms of productivity, each worker faces the same probability of *unemployment risk* if the firm decides to dismiss some workers. The unemployed agents devote their efforts to searching for a job. As stated above, we assume that the searchers do not know the types of the firms before they find employment. In this case, when a searcher happens to find a job, it is certain that he/she will accept the job. The equilibrium wage is determined by the so-called Walrasian auctioneer to clear the labor market.

Households maximize expected utility from consumption stream $\{c_t\}_{t=0}^{\infty}$ and minimize disutility from their search efforts $\{e_t\}_{t=0}^{\infty}$. The objective function is such that:

$$\max E \sum_{t=0}^{\infty} (\beta(1-\sigma))^t \left[u\left(c_t\right) + v\left(e_t\right)\right],$$

where $0 < \beta < 1$ is a discount factor, and σ is the probability of retirement, as defined in the previous section. The instantaneous utility function $u\left(c_t\right)$ is assumed to be strictly increasing, strictly concave, and continuously differentiable. Searchers devote their efforts e_t to finding a job, where the range of effort is normalized in the range $e_t \in [0, 1]$. The disutility of effort, $v(\cdot)$, is strictly decreasing and differentiable. By construction, the workers who do not search, receive no disutility from supplying their labor, i.e., $e_t = 0$ and $v(0) = 0$. Thus, workers do not quit their jobs for the purpose of leisure.

2.2.1 Workers

We divide a period into two subperiods, which are called *ex ante* and interim. Suppose a worker, who is hired by a firm of type (n_{t-1}, s_t) has some financial asset, $a_t \in \mathcal{A} \subseteq \mathbb{R}_+$. In period t, the *ex ante* value function of the worker is defined as follows.

$$\hat{W}\left(a_t, n_{t-1}, s_t\right) = \left(1 - J\left(n_{t-1}, s_t\right)\right) W\left(a_t, n_t, s_t\right) + J\left(n_{t-1}, s_t\right) \hat{S}\left(a_t, d_t = 0\right) \tag{3}$$

The *ex ante* value function of the worker is defined as the expected sum of an interim value function of the workers, $W\left(a_t, n_t, s_t\right)$, and an *ex ante* value function of the searchers, $\hat{S}\left(a_t, d_t\right)$. These functions are defined below, using the job-loss probability, $J(n_{t-1}, s_t)$. The function $J(n_{t-1}, s_t)$ determines whether the *ex ante* worker can work for the firm of type (n_t, s_t) during the period. A household agent who is dismissed at time t can search for a job in the same period.

Workers with state (a_t, n_{t-1}, s_t) face an unemployment risk with a probability of $0 \leq J\left(n_{t-1}, s_t\right) \leq 1$, and they may become searchers. The job-loss probability is endogenously determined by the decision of the firm.

$$J\left(n_{t-1}, s_t\right) = \max\left[0, 1 - \frac{g\left(n_{t-1}, s_t\right)}{(1-\sigma)n_{t-1}}\right]$$

If the firm by which a worker is employed decides not to dismiss workers, or if it decides to expand the size of the firm, then workers do not need to worry about unemployment risk, i.e., $J\left(n_{t-1}, s_t\right) = 0$. On the other hand, if an employer decides to reduce the size of the firm, some randomly chosen workers will be dismissed. In this situation, unemployment risks exist.

If an *ex ante* worker is not dismissed, he/she can choose levels of consumption c_t and savings a_{t+1} to solve the following dynamic programming problem:

$$W\left(a_t, n_t, s_t\right) = \max_{c_t, a'_{t+1}} \left\{u\left(c_t\right) + (1-\sigma)\beta E_t \hat{W}\left(a'_{t+1}, n_t, s'_{t+1}\right)\right\}, \tag{4}$$

subject to

$$c_t + a_{t+1} \leq \left(1 - \tau^{\mathrm{ui}}\right) w + \left(1 + r_t\right) a_t,\ \ a_t \geq 0,$$

where $W\left(a_t, n_t, s_t\right)$ is an interim value function of the worker with state $\left(a_t, n_t, s_t\right)$. Note that because firm size in the next period is already determined, $n_{t-1}$is replaced by n_t. Although the worker obtains labor income, part of his/her wage, $\tau^{\mathrm{ui}} w$, is taxed away by government as an unemployment insurance payment.

2.2.2 Searchers

Ex ante searchers in period t choose an effort level e_t to maximize the following expected value function:

$$\hat{S}\left(a_t, d_t\right) = \max_{e_t} \left\{-v\left(e_t\right) + p\left(e_t\right) \int \phi^{+}\left(n_{t-1}, s_t\right) W\left(a_t, n_t, s_t\right)(dn, ds) + \left(1 - p\left(e_t\right)\right) S\left(a_t, d_t\right)\right\}, \tag{5}$$

where d_t describes the duration of unemployment. If a searcher expends effort e_t, he/she can find a job with a probability of $p\left(e_t\right)$. As searches are undirected, the result is that the workers are randomly matched with employers who want to expand the size of their firm. That is, the workers face a size distribution of expanding firms, $\phi^{+}\left(n_{t-1}, s_t\right)$. If the searcher cannot find a job, he/she becomes interim unemployed.

If an *ex ante* searcher cannot find a job, he/she becomes a searcher and chooses consumption and savings to solve the following problem.

$$S\left(a_t, d_t\right) = \max_{c_t, a'_{t+1}} \left\{u\left(c_t\right) + (1-\sigma)\beta \hat{S}\left(a_{t+1}, d_{t+1}\right)\right\} \tag{6}$$

subject to

$$c_t + a_{t+1} \leq \mu\left(d_t\right) w + \left(1 + r_t\right) a_t,\ \ a_t \geq 0$$

The searcher receives unemployment insurance from the government, depending on his/her historical unemployment status, where $\mu\left(d_t\right)$ represents the replacement rate. The amount of unemployment insurance depends on the duration of unemployment, d_t. Denoting $d_t \in \{0, \ldots, D, D+1\} \equiv \mathcal{D}$, we assume that if d ranges between 0 and D, the worker is eligible for unemployment insurance, and that if $d = D+1$, the worker does not receive unemployment insurance. When the duration is zero, this implies that agents are unemployed just during that period, i.e., $\hat{S}\left(a_t, d_t = 0\right)$.

Let us define the policy function of interim workers as $a' = A^W(a_t, n_t, s_t)$, and that of interim searchers as $a' = A^S(a_t, d_t)$.

2.2.3 The Transition Law of the Households

Define a fraction (a probability measure) of workers with state (a, n, s) as $\psi^W(a, n, s)$, and searchers with state (a, d) as $\psi^S(a, d)$. Then, the distributions of households are defined as follows.

$$\psi^W(a', n', s') = (1-\sigma)\int_{\substack{A^W(a,n,s)=a',\\ g(n,s)=n'}} [1 - J(n,s)]\, Q(s', s)\, A^W(a, n, s)\, \psi^W(a, n, s)\,(da, dn, ds)$$
$$+ \sum_{d=0}^{D+1} \int_{A^S(a,n,s)=a'} p[e(a)]\, \phi^+(n', s')\, A^S(a, d)\, \psi^S(a, d)\,(da)$$

$$\psi^S(a', d') = \sum_{d=0}^{D+1} \int_{A^S(a,n,s)=a'} (1 - p[e(a)])\, A^S(a, d)\, \psi^S(a, d)\,(da) + \sigma I[a' = 0],\ d' = d + 1$$

The next period's workers are those who are not dismissed and those who were dismissed but can find a job. In the same way, searchers in the next period are those who still cannot find a job, including zero-duration searchers.

The aggregate supplies of labor and capital are completely determined by the consumers' behaviors. The labor supply, N^S, is endogenously determined by the fraction of agents who are workers. The aggregate capital, K^S, is an integration of workers' and searchers' distributions.

2.3 Government

Government collects unemployment insurance tax from workers and pays it to searchers who are eligible for the insurance:

$$w\tau^{\text{ui}} N^S = \sum_{d=0}^{D} w\mu(d)\, N^{\text{unemp}}(d), \tag{7}$$

where N^{unemp} is the number of agents who are unemployed with duration d. We assume that the government must balance the budget period by period.

2.4 The Definition of Recursive Equilibrium

Now, we define the stationary recursive equilibrium. Before examining the specifics of the definition, we explain the timing of events. At the beginning of period t, the idiosyncratic productivity shocks are realized. Given these shocks and the last period's employment level, employers decide whether to expand or reduce their firms. Then, the employment risks that workers face are endogenously determined. Some workers are dismissed and pooled as searchers with search duration periods of zero. Those searchers with search durations of zero start devoting their efforts to finding a job and are matched to firms that are expanding their size. In the stationary state, the number of agents who are fired is equal to the number of agents who find a job.

Definition 1 *A recursive stationary competitive equilibrium is a set of value functions* $(\hat{W}, \hat{S}, W, S)$ *and policy functions* $(A^W, A^S, e(a))$ *for households, a value function* V*, and a policy function* g *for firms, prices* (w, r)*, and an unemployment insurance* $(\mu(d), \tau^{ui})$*, a finite measure* (ϕ, ψ^W, ψ^S) *such that:*

1. *given prices* (w, r) *and unemployment insurance* $(\mu(d), \tau^{ui})$*,* $\hat{W}$ *solves the functional (3) and* $\hat{S}$ *solves the functional (5), and* (A^W, A^S) *are the associated policies;*

2. *given prices* (w, r)*, each firm's value function solves the functional (2), and the policy function* g *maximizes the value of the firm;*

3. *market clearing is as follows:*

$$N^D = N^S,\ K^D = K^S;$$

4. *the government determines a tax rate* τ^{ui} *to solve the equation (7); and*

5. *the stationary distributions are consistent with the behavior of the firms and households (a Law of Motion), and are invariant over time.*

The definition is quite standard, and our proof of the existence of the equilibrium follows Aiyagari (1994).

3 Calibration

3.1 Parameters of the Model

As it is impossible to solve the model analytically because of strict nonlinearity, we have to resort to numerical methods. Our target is the US economy. We compute a stationary recursive competitive equilibrium under several unemployment insurance systems and compare them to examine the effects of unemployment insurance on welfare.

We assume that one period in the model is equal to one quarter, and we set the parameters of the model to fit this length of time. We need to calibrate the period to be short to ensure that the duration of unemployment insurance matches the real economy. The probability of retirement, σ, is determined such that a household works for 40 years on average. To control an annual interest rate at a benchmark equilibrium to be around 4%, which is close to the 10-year Treasury note interest rate, we set the annual discount factor to be $\beta \approx 0.9962$. The instantaneous utility function is of logarithmic form and constructs an elasticity of intertemporal substitution equal to one for the short length of time. Following Alvarez and Veracierto (2001), we construct a functional form of the disutility of labor to be $v(e_t) = \Omega \frac{(1-e_t)^{\omega}-1}{\omega}$, $\Omega > 0$, $\omega > -1$, and the job-finding probability to be $p(e_t) = e_t^{\xi}$. Thus, the search activity is characterized by the disutility parameters (ω, Ω) and the curvature of the job-finding probability, ξ. Assuming a close-to-linear job-finding probability ($\xi = 0.98$) and a disutility function ($\omega = 0.98$), we set $\Omega = 11.0$ to match an equilibrium unemployment rate of the model ranging from the latter half of 4% in the benchmark case. As we calibrate the disutility function and the job-finding probability to be close-to-linear, the search effort is sensitive to changes in unemployment insurance. This sensitivity is described from evidence suggested by Meyer (1990) and adopted by Alvarez and Veracierto (2001). In a later section, we show that the formulation plays a significant role in determining the welfare effect of unemployment insurance.

Following previous research, such as Cooper, Haltiwanger, and Willis (2004) and Khan and Thomas (2003), we assume that the logarithms of the idiosyncratic productivity shocks follow an AR(1) process as follows.

$$\ln s_{t+1} = \rho \ln s_t + \varepsilon_t,\ \varepsilon_t \sim \mathcal{N}\left(0, \sigma_{\varepsilon}^2\right) \tag{8}$$

In this form, the productivity shocks are characterized by the persistence ρ and the variances, σ_{ε}^2. From Table 2*a* in Cooper et al. (2004), we set the annual persistence of the shocks to be $\rho = 0.7$ and adjust it for the quarterly period length. Cooper et al. (2004) estimate structural parameters—such as the persistence, the variances, and coefficients of several cost functions in plant-level dynamics—from the method of simulated moments by exploiting the Longitudinal Research Database.[8] The value of $\rho = 0.7$ is an average of several estimations. For our purpose, we need to calibrate the unemployment risks carefully to match the JCR/JDR in the US. The variances of the shocks are selected to match the JCR/JDR of the model at equilibrium in line with the observation of Davis et al. (1996). Using new data sources developed by the Bureau of Labor Statistics and the Bureau of the Census, Davis, Faberman, and Haltiwanger (2005) estimate the quarterly rate of job creation and destruction and conclude that the value is higher than that previously considered, reaching around 10% in the private sector and 6% in manufacturing. We choose $\sigma_{\varepsilon} = 0.056$, which leads to a quarterly JCR/JDR of around 5.48%. From this parameterization, we approximate the AR(1) process to be a nine-state Markov chain using Tauchen's (1986) method.

[8] Cooper et al. (2004) consider several cost functions such as quadratic, fixed, and disruption cost forms. The persistence of the shocks changes for several cases from $\rho = 0.48$ to $\rho = 0.996$, as reported in Table 3*a*. For simplicity, we fix $\rho at 0.7$, as in Table 2*a*. Cooper et al.'s estimation of the volatility parameter σ_{ε} suggests that it may be much higher than ours. However, such a high value would overestimate the JCR/JDR in our model.

To obtain a stationary distribution of firms' size, we have to assume that the production function we employed has decreasing-return-to-scale technology. Cooley, Marimon, and Quadrini (2004) set the span of control, which is defined by $1 - \theta - \eta$, to be 0.15.[9] In other words, the share of national income going to owners of establishments is 15%. From Cooley et al. (2004), we set $\eta = 0.25$, which implies that the labor share of the output is 0.6. The depreciation rate δ is employed from research by Cooper and Haltiwanger (2005).

We collect all calibrated parameters, which are adjusted to represent quarterly values, in Table 1.

3.2 Specification of the Unemployment Insurance

Following Alvarez and Veracierto (2001), the unemployed households are eligible to receive unemployment insurance for one quarter as a benchmark, namely $D = 0$. In our model, this assumption implies that the unemployed can receive insurance when they have been dismissed in period t and cannot find a job during this period. The replacement rate of the unemployment insurance in the benchmark case is set to be 0.5, following Young (2005). Based upon this benchmark, we want to examine several unemployment insurance systems. First, we change the replacement rate from 0 to 0.5 to investigate the sharing effects of the insurance. Second, we consider a case where the government enhances the unemployment insurance opportunity for one period. In other words, unemployed agents can receive unemployment insurance for two periods. Third, we consider an alternative method of paying the unemployment insurance. Although the total amount of the unemployment insurance remains the same, we divide it over two periods, rather than just one period.

The cost of job destruction is based on Alvarez and Veracierto (2001), namely one month, i.e., $\tau^f = 0.33$. As is well known, such a cost function makes the corresponding policy function of the (s, S)-type. Although Alvarez and Veracierto (1998) assume that the costs of job destruction are paid to unemployed agents as severance payments, we neglect those payments because we want to focus on the pure effect of the unemployment insurance.[10] In sum, Table 2 reports all the policies we considered.

[9] Atkeson and Kehoe (2001) and Atkeson, Khan, and Ohanian (1996) also assume a decreasing-return-to-scale technology.

[10] Alvarez and Veracierto (1998) carefully study the risk-sharing effect of the severance payments.

4 The Welfare Effects of Unemployment Insurance on the General Equilibrium

4.1 Aggregate Statistics and Social Welfare

First, we discuss the aggregate statistics of the computed results under several replacement rates. In our model, workers face unemployment risks that are endogenously generated by firms' decisions under idiosyncratic production risks. As new jobs are created by the decisions of the firms, it is not possible for all unemployed agents to find a job even if they make enthusiastic search efforts. This environment differs from that of several other research studies of unemployment insurance models in that we consider the demand side of the labor market explicitly. With unemployment insurance, unemployed agents are not forced to decumulate their assets, but they can survive on the insurance payments from government. However, some workers hold sufficient assets to hedge their unemployment risks, and they may not want to pay a tax as insurance. The search effort of the unemployed agents depends upon their level of asset holdings. Therefore, the endogenously generated wealth distribution significantly affects the impact of unemployment insurance on labor and capital markets at general equilibrium.

In Table 3, we report the computed results at the equilibrium for each replacement rate and unemployment insurance policy. The results are considered in terms of the impact upon interest rates, unemployment rates, aggregate capital, tax rates, Gini coefficients, and certainty equivalent (CEQ) consumption levels.[11] For the purpose of comparing the welfare consequences of different types of unemployment insurance, we define the social welfare of the economy SW_0 as follows.

$$SW = \int \hat{W}\left(a,n,s\right)\psi^{\hat{W}}\left(a,n,s\right)\left(da,dn,ds\right) + \sum_{d=0}^{D+1}\int \hat{S}\left(a,d\right)\psi^{\hat{S}}\left(a,d\right)\left(da\right) \tag{9}$$

$$SW = \frac{1}{1-(1-\sigma)\beta}\ln\bar{c} \tag{10}$$

From this welfare criterion, the CEQ consumption level, $\bar{c}$, is calculated from inversion of the function in Equation (10). Although we have excluded firms' profit from the welfare criterion, we will compare the aggregate output and profit in the next section.

We confirm that the numerical results of our model in the benchmark replicate the US economy well.[12] We have assumed that all unemployed workers are eligible for unemployment

[11] We omit the consideration of equilibrium wages because the level of wages is not relevant for our purpose. All rates of change in the wage for each replacement rate are shown in Table 4.

[12] In our computation, gaps between demand and supply in labor and capital markets, and gaps in the budget balance of the government are below 0.1% at equilibrium for all cases. Thus, we can say that the model succeeds in computing the equilibrium.

insurance, which implies that the equilibrium unemployment rate tends to be high. Nevertheless, the unemployment rate matches that of the real economy well. The yearly interest rate is about 4%. One of the most important target variables for our calibration is the JCR/JDR rates, which are about 5.48% for all cases. As noted above, several papers report results that conflict with the *quarterly* JCR/JDR rates. For example, Davis et al. (1996) report that the quarterly JCR/JDR rates are approximately $5-6\%$ in Table 2.1 in Davis et al. (1996). In contrast, using another dataset, Davis et al. (2005) show the value of the rate to be nearly 8% of employment in the private sector. For our purpose, we want to endogenize the unemployment risks, and we believe that the calibrated JCR/JDR can explain the economy in the United States well without loss of generality.

As expected, the unemployment rate decreases from 5.46% to 2.54% when the replacement rate is reduced by government from the benchmark case to zero. Our calibration of the disutility of the search effort implies that the effort function over assets is highly intensive near zero assets. When households face unemployment risks in incomplete markets, they share such risks in two ways: they devote more effort to searching for a job, and they make precautionary savings. If a replacement rate is not high, all agents tend to hold more wealth because of strong precautionary motives. This implies that the wealth distribution shifts right. In contrast, given a certain level of wealth holdings, a searcher wants to devote more effort to job search. Because the search effort is a decreasing function of wealth, the distribution of endogenous asset holdings and the strength of the job search effort have opposite consequences for the equilibrium unemployment rate, which complicates the welfare effect of the unemployment insurance. From Table 3, it is apparent that the aggregate capital increases and the corresponding tax burden decreases when the replacement rate is reduced. We could not find any relationship between the replacement rate and the Gini coefficients.[13]

4.2 A Comparison under Several Replacement Rates

In Table 4, we compare rates of change of CEQ consumption levels between the benchmark case and cases for which the replacement rate varies from 40% to 0%. In addition, Table 4 reports the percentage changes of the interest rate, the wage, disposable income, and aggregate output. As the literature on the optimal replacement rate suggests, even if the change improves households' welfare as measured by the CEQ consumption, the welfare gain is small, below 1%.

In Table 4, we show that the certainty equivalent consumption of the social welfare defined in Equation (10) reaches its highest value at 0% ($\mu(0) = 0.0$) of the replacement rate. Alvarez and Veracierto (1998) made the same finding under a similar setup, although we have modified

[13]It is well known, following Aiyagari (1994), that the Gini coefficient of this class of models is usually low compared with an actual value of over 0.7. However, it loses no generality for our investigation.

two aspects of their model in relation to search timing and bequests. For instance, by decreasing the replacement rate from the benchmark case of $\mu(0) = 0.5$ to 0.4, the CEQ consumption increases to 0.235%. Within our calculated range, the replacement rate of 0% yields the highest social welfare. In other words, an average household benefit of 0.767% measured by the CEQ consumption is yielded by changing from the benchmark to the optimal policy regime.

The equilibrium wage decreases when a low unemployment rate leads to a reduction of the replacement rate. On the other hand, as reported in Table 4, the corresponding interest rates do not change so strongly. This implies that the main general equilibrium effect of increasing the replacement rate is the easing of the search effort. That is, because of the upper bound of labor demand determined by the firms and the large elasticity of the hazard rate with respect to the replacement rate, the labor market at equilibrium adjusts more sensitively via the wage than does the capital market. Because a reduction of the replacement rate results in a high job search intensity, the aggregate labor supply must increase. However, firms' demand for labor is less sensitive than supply in response to the change of policy and factor prices. Nevertheless, the gap in the labor market should be adjusted via wages.

Inclusion of the capital market and savings complicate the above mechanisms. If the replacement rate is decreased, all agents want to save more owing to precautionary motives. Then, the aggregate capital supply increases and the interest rate falls, which implies that the firms obtain greater profits from a lower factor price. In our model, extensive activities by the firms expand their demand for labor . However, the supply of labor, owing to eager searchers making strong job search efforts, exceeds the expansion of labor demand. Therefore, the equilibrium wage computed in Table 4 decreases. These mechanisms do not appear in the model with a representative firm, and precautionary saving motives have a positive role for production. Aggregate output and profit increase according to the reduction of the corresponding replacement rate because the low interest rate and wage allow the firms to expand their economic activities. As a result of the wage reduction, tax rates decrease rapidly. Thus, disposable income, defined as $(1 - \tau^{\mathrm{ui}})w$, increases as a result of a reduction in the replacement rate.

When the aggregate output is high and the unemployment rate is low, social welfare is high in general. However, agents are heterogeneous. Expanded aggregate output enhances aggregate consumption opportunities, which means that expected utility usually increases for large numbers of households. However, because very poor agents receive a high marginal benefit from unemployment insurance, a zero replacement rate is not necessarily optimal. Agents do not receive equal rewards from the expansion of economic activity. Although this explanation seems to be intuitive, some papers do not find that the optimal replacement rate is positive. In fact, Young (2005) finds the optimal rate to be zero using a dynamic model with search efforts and capital accumulation. Young's result is consistent with research by Alvarez and Veracierto (1998), who found that if there are sufficient asset holdings, the optimal replacement rate is exactly zero. In addition to the social welfare (an average agent's utility defined in Equation (9)), we computed the expected values of workers and searchers. The optimal replacement rate for a worker is not zero but 30%. In our model with $D = 0$, the expected value of the *ex ante* worker includes the *ex ante* searchers' value function. Thus, the welfare effects of the unemployment insurance are reflected in the worker's CEQ consumption.[14] Therefore, although the average social welfare, which is usually used as a measure of welfare in the literature, is highest at a zero replacement rate, we need to investigate carefully the welfare effect of the unemployment insurance.

One interesting finding to note is the policy of dividing the unemployment insurance between two periods, i.e. from $\mu = \{0.5, 0.0\}$ to $\mu = \{0.25, 0.25, 0.0\}$. Although people who experienced unemployment for two periods can receive the same amount of payments from the government, the corresponding CEQ consumption increases.

5 Unemployment Insurance and Asset Distribution

5.1 Unemployment Insurance for Several Asset Classes

Recent literature on precautionary savings shows that agents hold more assets for precautionary motives when they are faced with idiosyncratic income risks. In relation to the agents, our model is similar to such research, although the income risk is endogenously determined in our model. If an agent holds sufficient wealth, he/she may not want to make insurance payments to the government. In particular, the tax burden is severe for workers who hold very few assets, because a large fraction of their total income is labor income. In other words, the fraction of the tax burden as a proportion of total wealth is high for the poor agents. Hence, once the agents find a job, they may not want to make such tax payments. Indeed, labor income is a scarce resource for those who have few assets. In contrast, unemployed agents with nearly zero assets are anxious to receive insurance from the government. Therefore, the unemployment insurance has a sharply contrasting effect on workers and searchers with nearly zero asset holdings. Even if aggregate economic activity is expanded by lowering the replacement rate, those who have few assets may suffer from such a policy change. In our setup, in addition to those features, we need to consider disutility from search effort for each asset level, the influence of which is not usually considered in the precautionary saving models. Moreover, since the factor prices are also changed for each equilibrium through savings decision *and* effort, the change of unemployment insurance system has different effects on several asset levels of workers and searchers.

[14] The searcher's CEQ consumption increases when the replacement rate is reduced, because the left-hand side of the distribution used to compute the expected utility lessens and the right-hand side of the distribution tends to be overestimated if the replacement rate is low.

We have computed the CEQ consumption, which is calculated from the value function of workers and searchers, for some asset grid points. The evaluated asset grid points are chosen for each 5 percentile point between 0% and 95%.[15] We calculate the evaluation grids for each percentile point of the cumulative asset distribution in the benchmark model and compute the CEQ consumption at the points. That is, initially, we do not consider an endogenous change of the wealth distribution. Later, we will include an endogenous change of the evaluation grids, as such an experiment indicates whether a change of the replacement rate affects the welfare of agents given their asset level and determines the long-run effects of the policy change. The results are presented in Figures 1 and 2. In those figures, we represent the rates of change between the CEQ consumption with zero replacement rate and that with the other replacement rates for each asset grid. For example, 0.2% on the vertical axis indicates that changing the replacement rate from 0% to another replacement rate generates a consumption gain measured by the CEQ consumption at the same level of asset holdings.

First, we find that increasing the replacement rate generates weakly U-shaped effects over the assets for workers. Workers' welfare improve for almost all asset levels following the incremental introduction of unemployment insurance. This is particularly the case when $\mu(0) = 0.2$ and 0.3. We note that the zero percentile grid indicates zero assets for all cases.[16] The workers with zero assets can benefit from a marginal increase of the unemployment insurance. Thus, the introduction of the insurance creates great benefits for the workers with zero assets, because the welfare of those agents is improved by a marginal increase of the value function of the searchers with a zero search duration, $\hat{S}(a_t, d_t = 0)$, as in Equation (3). With moderate amounts of insurance payments, zero-asset consumers need not devote too much effort to job searching even when they are dismissed. On the other hand, the welfare of workers with asset holdings that are low but above zero (e.g., 5% in the Figure) does not improve relative to those with zero assets. Although the unemployment insurance yields benefits for the agents with near-zero assets, the tax burden begins to impact upon them, and they cannot receive rewards from the high interest rate.

The equilibrium interest rate (and the wage) increase as the replacement rate becomes higher,

[15] Krusell and Smith (2002) investigate the cost of the business cycle for different groups classified by their asset levels.

[16] As shown in Table 4, social welfare is maximized at $\mu(0) = 0.0$. In contrast, Figure $1-4$ reports that a wide range of agents benefit when the replacement rate is increased from zero. Those contradictions are explained by the unemployment rates and the endogenously generated distributions used to compute the expected utility. Generally, the expected utility of searchers is lower than that of workers. Hence, if the fraction of workers decreases (i.e., the unemployment rate increases), social welfare also decreases. Moreover, as the proportion of low asset holders is relatively low in the case of a low replacement rate, the expected value of the searchers is overestimated. The U-shaped curves are highest when the replacement rate is 30% because the workers' CEQ consumption is optimal at that value. This is consistent with Table 4.

as shown in Table 4. In addition, wealthier agents do not want to devote great efforts to job search. Therefore, the wealthier agents reap a greater welfare gain from a high replacement rate than do those agents with low assets. Because the searchers in Figure 2 have already experienced one period of unemployment after losing their jobs, the change of the replacement rate has little effect on the zero asset holders. The upward-sloping curves arise from an increase of the interest rate and the reduction of onerous search efforts. In addition, the value function reflects the benefit of not having to move back and forth so much between employment and unemployment. Therefore, the introduction of unemployment insurance benefits agents with zero assets and wealthier households.

Second, we include an endogenous change of the wealth distribution in Figures 3 and 4. In other words, the evaluation grids change. For workers without high percentile points, the CEQ consumption increases, as in the previous explanation. However, the slope of the curve is moderate, or even flat. When endogenous changes of the distribution are included, an increase in the replacement rates mean that the high wealth holders come to hold relatively fewer assets, as confirmed in by our numerical computation. Thus, agents with high percentile points benefit less they did in the case without endogenous changes of the wealth distribution from the change of the insurance policy. Relatively wealthier agents do not want to invest in search efforts owing to our assumption of a highly sensitive disutility function and a linear job-finding probability. Hence, those who have sufficient wealth do not suffer from the disutility of search effort.

To conclude, through equilibrium effects, the introduction of unemployment insurance generates moderate benefits for wide level of assets. In particular, low wealth holders benefit but wealthier agents are disadvantaged in the long run. Although we cannot explicitly consider the transition path away from an unemployment insurance regime to another one, these experiments indicate the short-run and long-run effects of changing unemployment insurance. Although increasing the replacement rate benefits each percentile point of asset holdings, the welfare gain is not high when endogenous change of the wealth distribution is taken into account. In the long run, near-zero-asset agents benefit from the introduction of unemployment insurance, but the wealthier agents are disadvantaged by the endogenous wealth change.

5.2 Constant Search Efforts and Unemployment Insurance

5.2.1 Aggregate Results

Our specification of the disutility of effort and the job-finding probability implies that the job-finding rate over assets is sensitive to changes in the replacement rate, and also to changes in the levels of asset holdings.[17] If the search effort is sensitive to a change of the replacement rate, such a policy change is mainly adjusted by the effort. Hence, changing the replacement rates reflects an equilibrium unemployment rate and the wage in the labor market. On the other hand, the interest rate does not change much. Such a specification of the disutility of effort is empirically estimated from the hazard function. However, using Dutch data with a subjective reservation wage, Bloemen and Stancanelli (2001) show that, even if the employment probability is negatively correlated with overall wealth, the elasticity is low. If the search effort is not sensitive to the change of asset holdings, then the welfare consequences of our results must change significantly. In this section, we consider the case of inelastic search intensity. For simplicity, we assume a constant search effort, which is the average of the benchmark computed above.

In Table 5, we assume that search effort is constant for each level of asset holdings and is set at $e = 0.517$, which is an average of the benchmark model.[18] Changes of factor prices indicate changes in the demands for labor and capital, as determined by the firms. If the effort is constant over assets, the unemployment rate does not decrease much when the replacement rate is reduced. In such a case, households make provision against unemployment risks only by accumulating wealth. As a result, the aggregate capital increases compared with the flexible search model. In the constant search economy, all agents want to share the risks by accumulation of wealth. However, a strict upper bound on labor supply leads to low demand for capital by the firms. As a consequence of strong precautionary motives, the firms' production does not expand much even though the interest rate declines. Nor does social welfare increase much when the replacement rate is reduced. We cannot find any clear relationship between the CEQ consumption and social welfare in Table 5. However, this is not significant because the constant effort is a simplification that does not involve optimal behavior by the agents.

From an assumption regarding liquidity constraints, the asset levels cannot fall below zero. Strong precautionary motives lead agents with low or middle levels of assets to hold more wealth when the replacement rate is low. In contrast, wealthier agents do not wish to accumulate their wealth as much because the interest rate is low. Therefore, decreasing the replacement rate implies a concentration of assets, as suggested by the Gini coefficients in Table 5. In the flexible search model, if agents hold more wealth, they need not devote so much effort to the disutility generating job search. Therefore, rich people have an incentive to accumulate their wealth even if the interest rate is low.

[17] This point was made by Alvarez and Veracierto (1998), who use calibration parameters estimated by Meyer (1990).

[18] An exception is agents with zero assets and searchers without insurance who, by assumption, must devote full effort to job searching.

Because a change in the unemployment insurance policy does not drastically change the unemployment rate, households react to the risks of unemployment through accumulation and decumulation of their assets. The unemployment insurance system obviously influences the interest rate in the capital market. We find that our assumptions on search behavior have different implications for households with different levels of assets.

5.2.2 Asset Distribution

The case of constant effort results in different consequences for CEQ consumption for benchmark asset levels.[19] If search effort is flexible, households with nearly zero assets will maximize their expected utility by increasing their efforts. Without such a mechanism, the welfare effects for zero-asset agents, whether they are workers or searchers, change significantly when the replacement rate is changed. Figure 5 indicates that workers with zero assets cannot receive benefits when the replacement rate is increased. Because the *ex ante* value function includes the *ex ante* value of searchers with zero-period unemployment experiences, as in Equation (3), it may be possible to increase the welfare of the zero-asset agents by incrementally increasing the replacement rate, as shown in Figure 1. However, such a mechanism vanishes when the search effort is constant. It indicates that the welfare effects of the unemployment insurance, described in the previous section, are achieved mainly through the adjustment of job search efforts. If the workers with near-zero assets are dismissed, they should devote a high level of effort to finding a job. Although such behavior brings disutility, a short length of unemployment is more beneficial than a long period of unemployment. Therefore, if the search effort is insensitive to asset holdings, the high replacement rate has no beneficial welfare effects for workers on the evaluation grids. In particular, the welfare loss is large for agents with near-zero assets. As we have employed an average effort level, agents with relatively high levels of wealth suffer from too great an effort level. Thus, the effects of a high interest rate are offset by the disutility of the job search effort. Searchers with zero assets receive a large benefit from the introduction of unemployment insurance (Figure 6). In contrast, workers with zero asset holdings cannot gain additional welfare because of the negative effects of insurance tax on their labor income.

Figures 7 and 8 indicate that including endogenous change of the wealth distribution results in slightly different effects for each agents with several assets. In particular, rich people who are positioned over the 90th percentile of the wealth distribution can benefit more than they did in the case without endogenous change of the wealth distribution when replacement rates are increased. Agents with high wealth holdings yield more income owing to the high interest rate. As stated above, if the search effort is constant, the right-hand side of the wealth distribution

[19]The benchmark evaluation points are the same as in the previous section.

is enlarged when the replacement rate is increased, which contrasts sharply with the case of the flexible search effort model.

This experiment seems to be unrealistic. However, we believe that there is a possibility that the job search efforts of searchers may hit an upper bound, e.g., search time does not exceed 24 hours. If a large fraction of the searchers face such a constraint, our numerical exercise indicates that the optimal replacement rate differs considerably depending on whether an average agent is considered or whether agents are assumed to have different levels of asset holdings. If the search effort is inelastic over assets, the insurance effect of the unemployment insurance is weakened. Moreover, the welfare consequences of the unemployment insurance are different for poor and rich people.

6 Concluding Remarks

In this paper, using the general equilibrium model with heterogeneous agents and search under establishment dynamics, we have shown that the effects of unemployment insurance differ considerably when an average agent is considered compared to the case where agents are assumed to have differing levels of asset holdings. Although social welfare measured by an average agent's utility is maximized at the zero replacement rate, workers can benefit from the introduction of unemployment insurance. Moreover, as intuition indicates, the insurance effect is strong for agents with near-zero asset holdings. We revealed that increasing the replacement rate has a large impact on low asset holders and on agents with high levels of wealth through the changes in the interest rate and the search effort. Although agents with zero assets receive a significant welfare gain when the replacement rate is increased, the optimal replacement rate is not positive when we use a welfare measure based on the average value of all agents. Reducing the replacement rate leads to greater economic activity by firms as a result of lower prices for labor and capital.

In addition, we revealed that when considering the effects of the unemployment insurance, the elasticity of the search effort over assets has significant implications for the macroeconomy and for different groups of agents. Even if agents hold almost zero assets, the introduction of unemployment insurance may not benefit them if the search effort is constant. This experiment implies that if the unemployed agents face an upper limit of effort and they inelastically devote that effort level regardless of the amount of assets they hold, then increasing the replacement rate has different effects for each group of asset holders. It is usually difficult to test empirically whether the search effort is sensitive to the asset holdings of agents, as the only thing we can observe is whether they find a job. Even though poor agents eagerly search for a job, the hazard rate over assets may not be different. In such a case, the insurance result will be different from the result in the case of flexible search, especially for agents with zero assets. Moreover, if we empirically test the relationship between the asset distribution and the hazard rate, an endogenous correlation between low wealth and the ability (or human capital) of an agent remains a considerable problem. Those who have a low level of wealth may be people with low skills, which implies that those agents will not find it easy to obtain another job if they are dismissed. The particular form of the search effort, or the relationship between savings and the reservation wage, is a subject for future research.

A Technical Appendix

This appendix explains details of the algorithms employed in this paper. Although the approach is basically the same as in Aiyagari (1994), we need to apply some particular techniques to solve it.

The computation steps taken are as follows. (1) Guess an interest rate and wage. Although our model contains three markets, those for consumption goods, capital, and labor, from Walras' law, we can omit the consumption goods market. Hence, we must find the equilibrium interest rate and wage. (2) Solve the firms' problem. In particular, we want to obtain policy functions for labor demand, $n_t = g(n_{t-1}, s_t)$. The idiosyncratic productivity process is approximated to a finite Markov chain of 9-state by Tauchen's (1986) method. The number of grids for n_{t-1} is 201. In order to match the job creation and destruction rates of the model to the data (the target value is about 5.5%), we have computed those values in the program. By integration of the firm size distribution, we obtain aggregate labor and capital demand. (3) Given the firms' policy functions, we solve the agents' dynamic programming problem in Equations (3) and (5). To economize on the grid numbers for the agents' problem, we neglect grids (n, s), which measure zero, even though these exist in the domain. To solve the problem rapidly, we combine the value function iteration and Howard's improvement algorithm, and the value function is approximated by a quadratic function. We take 50 points for the assets. (4) Obtaining the policy functions of firms and agents, we compute a stationary distribution of the model by forward-looking iteration. The number of asset grids is 500, and the policies not on the computed grid in the above computation are approximated by the cubic spline. (5) From integration using the distribution function, we can obtain the aggregate capital and labor supply. Comparing demand and supply, we find equilibrium by the bisection method.

Although our model is based upon the model developed by Alvarez and Veracierto (1999), without entry/exit decision of firms, we cannot apply their numerical algorithm. For market clearing, we need to find equilibrium prices of both the wage and the interest rate. These computations were applied by Alonso-Borrego et al. (2005).

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Discount Factor	β	0.985	(0.9962)
Disutility of Effort (Constant)	Ω	11.0	–
Disutility of Effort (Curvature)	ω	0.98	–
Job-Finding Probability	ξ	0.98	–
Retirement Probability	σ	0.025	(0.00625)
Capital Depreciation	δ	0.069	(0.01725)
Capital Share	θ	0.25	–
Labor Share	η	0.60	–
Persistence of the Shocks	ρ	0.70	(0.9147)
Standard Deviation of the Shocks	σ_ε	0.056	–

Table 1: Yearly Calibrated Parameters (Quarterly values in parentheses)

Firing Cost	τ^f	0.33
Hiring Cost	τ^h	0.0
Unemployment Insurance Plans	$\mu(0,1)$	$\{0.5, 0.0\}$
	$\mu(0,1)$	$\{0.4, 0.0\}$
	$\mu(0,1)$	$\{0.3, 0.0\}$
	$\mu(0,1)$	$\{0.2, 0.0\}$
	$\mu(0,1)$	$\{0.1, 0.0\}$
	$\mu(0,1)$	$\{0.0, 0.0\}$
One-Period Extension	$\mu(0,1,2)$	$\{0.5, 0.5, 0.0\}$
Divide UI	$\mu(0,1,2)$	$\{0.25, 0.25, 0.0\}$

Table 2: Unemployment Insurance Plans

	(Benchmark) $\mu(0) = 0.5$	$\mu(0) = 0.4$	$\mu(0) = 0.3$	$\mu(0) = 0.2$	$\mu(0) = 0.1$	$\mu(0) = 0.0$	divide UI	One-Period Extension
interest rate (%)	1.090	1.090	1.090	1.089	1.089	1.089	1.088	1.091
unemployment rate (%)	5.46	4.80	4.09	3.50	2.98	2.54	4.83	8.65
aggregate capital	3.83	3.85	3.87	3.89	3.91	3.92	3.85	3.72
tax rate (%)	1.89	1.26	0.75	0.40	0.15	0.00	0.94	3.89
Gini coefficient	0.547	0.546	0.545	0.545	0.546	0.548	0.539	0.544
CEQ consumption	0.180	0.181	0.181	0.181	0.181	0.182	0.181	0.178

Notes: The duration of the unemployment insurance is one period of the model. The interest rates are quarterly values. The job creation and destruction rates computed from the model are approximately 5.48%. As we focus on stationary states, the job creation and destruction rates excluding retirements must be equal.

Table 3: Equilibrium Results for Each Replacement Rate

	(Benchmark) $\mu(0) = 0.5$	$\mu(0) = 0.4$	$\mu(0) = 0.3$	$\mu(0) = 0.2$	$\mu(0) = 0.1$	$\mu(0) = 0.0$	divide UI	One-Period Extension
interest rate	0.0	−0.050	−0.084	−0.147	−0.172	−0.169	−0.219	0.080
wage	0.0	−0.115	−0.228	−0.346	−0.470	−0.551	−0.097	0.672
disposable income	0.0	0.530	0.931	1.175	1.296	1.369	0.878	−1.376
CEQ consumption	0.0	0.235	0.678	0.681	0.700	0.767	0.379	−0.962
CEQ consumption (worker)	0.0	0.136	0.490	0.420	0.384	0.411	0.224	−0.595
CEQ consumption (searcher)	0.0	1.324	2.972	4.353	5.779	7.216	2.622	−2.488
aggregate output	0.0	0.577	1.168	1.657	2.047	2.425	0.612	−2.628
aggregate profit	0.0	0.490	0.973	1.494	2.017	2.350	0.529	−2.695

Notes: These values are the relative differences between the value of the benchmark and that of each replacement rate. Disposable income is defined to be $(1 - \tau^{ui})w$. All values are percentile points.

Table 4: Equilibrium Comparison

	(Benchmark) $\mu(0) = 0.5$	$\mu(0) = 0.4$	$\mu(0) = 0.3$	$\mu(0) = 0.2$	$\mu(0) = 0.1$	$\mu(0) = 0.0$
interest rate (%)	1.089	1.088	1.088	1.087	1.087	1.086
aggregate capital	3.832	3.830	3.834	3.837	3.841	3.860
Gini coefficient	0.489	0.488	0.486	0.484	0.481	0.478
interest rate differences	0.0	−0.038	−0.084	−0.134	−0.197	−0.218
wage differences	0.0	0.008	−0.002	−0.008	−0.023	−0.063
CEQ consumption differences	0.0	0.028	0.059	0.109	0.105	0.179
CEQ consumption (worker)	0.0	0.030	0.061	0.102	0.085	0.062
CEQ consumption (searcher)	0.0	−0.063	−0.105	−0.103	−0.105	0.478

Notes: The five lower rows are the relative differences between the benchmark and each case of the replacement rate. The values are percentile points. The interest rates and the unemployment rates are all quarterly values.

Table 5: Equilibrium Results for Each Replacement Rate

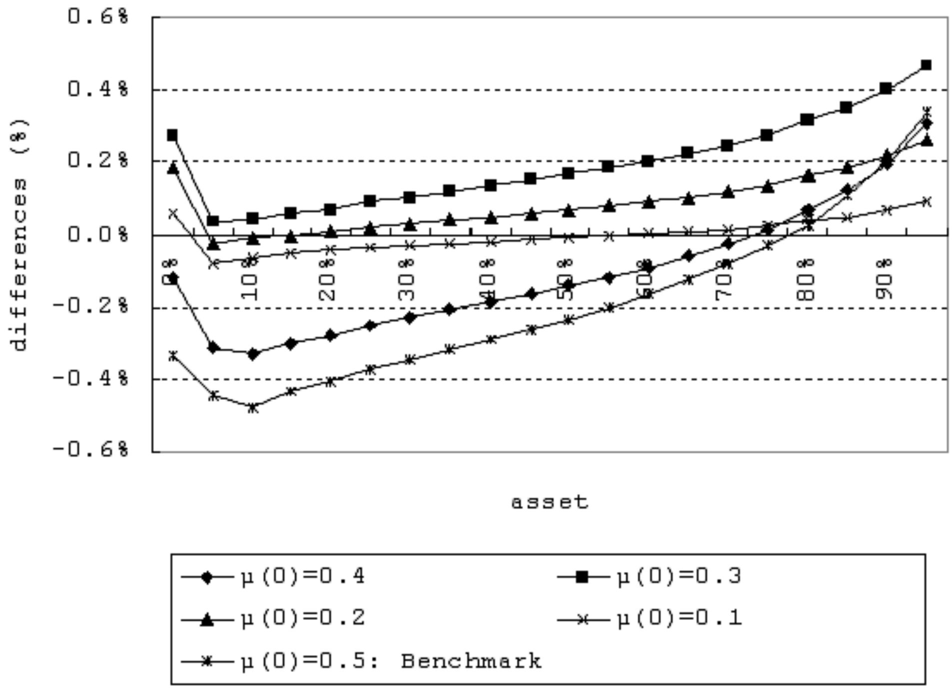

Figure 1: CEQ Consumption on Benchmark Asset Points (Worker)

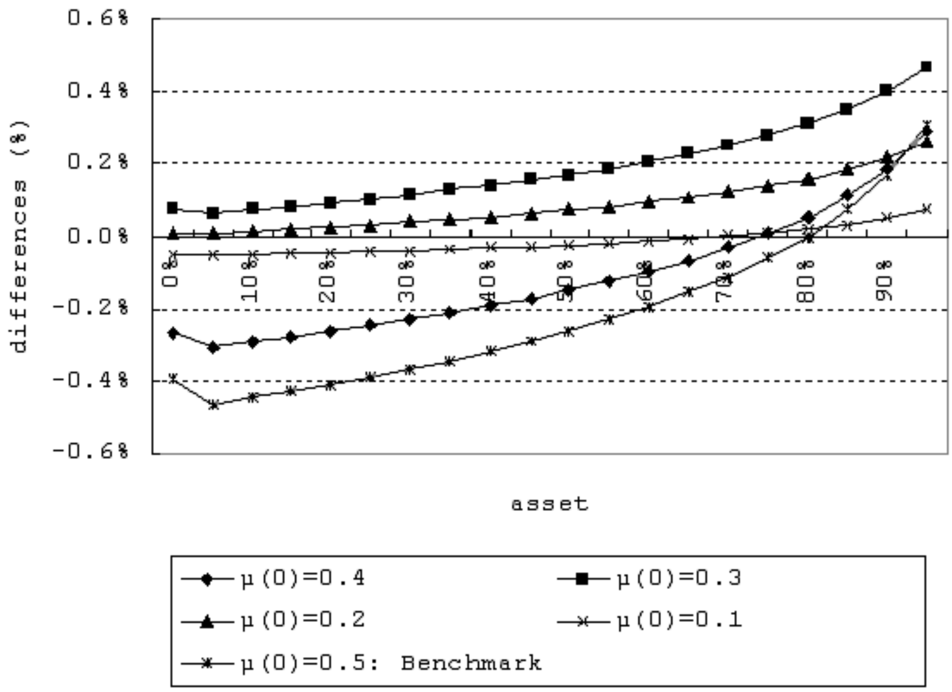

Figure 2: CEQ Consumption on Benchmark Asset Points (Searcher)

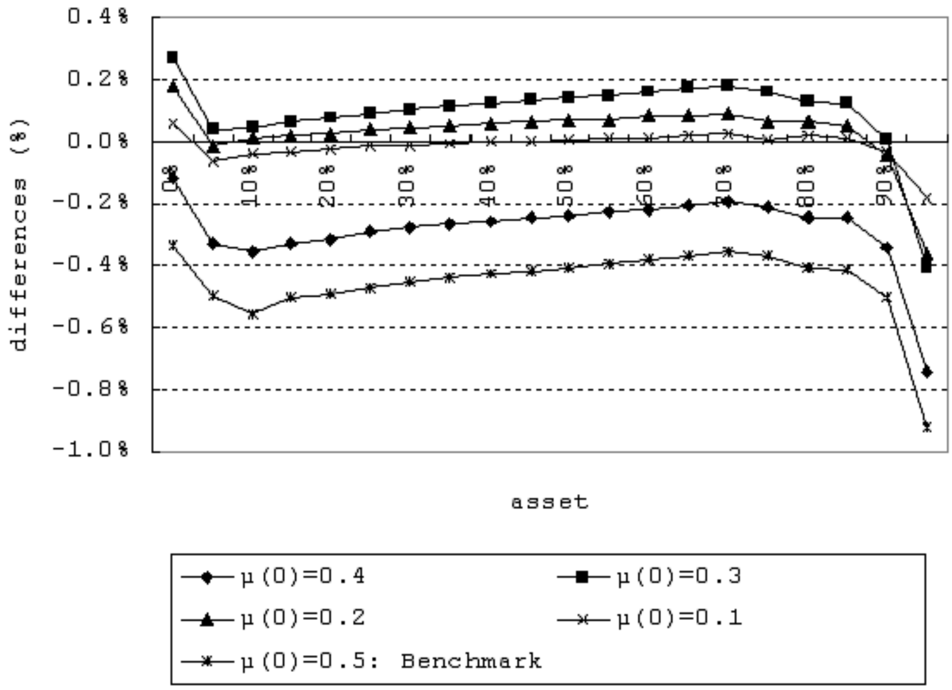

Figure 3: CEQ Consumption for Each Class of Asset (Worker)

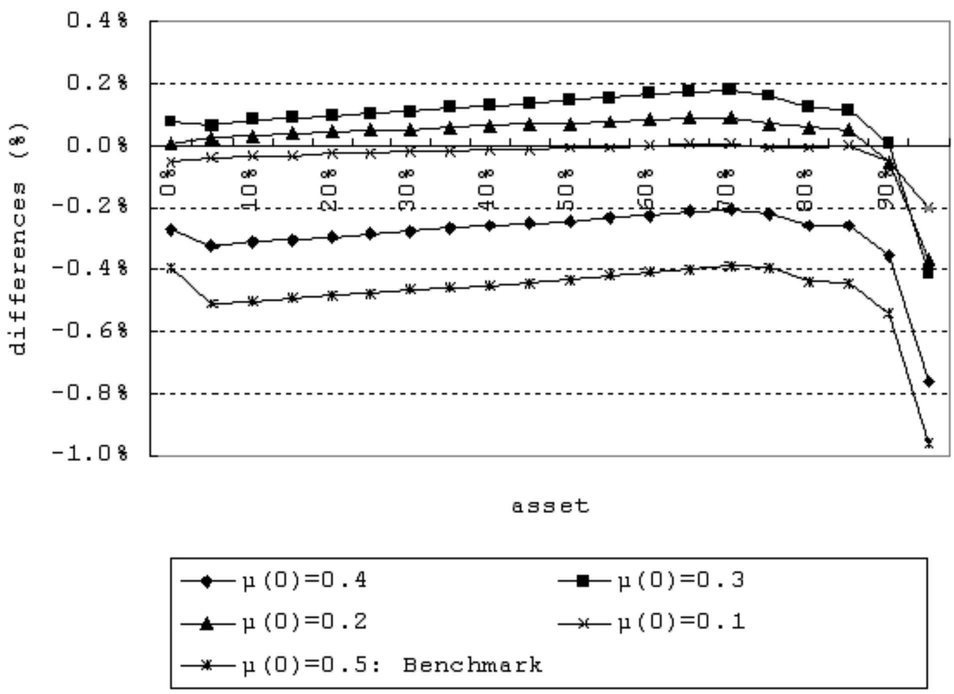

Figure 4: CEQ Consumption for Each Class of Asset (Searcher)

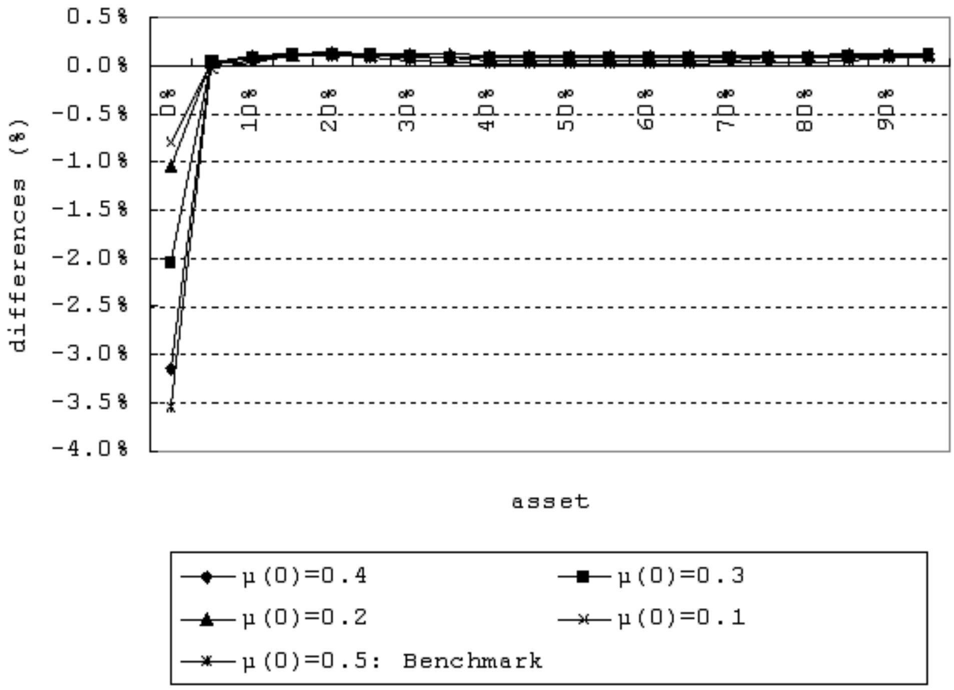

Figure 5: CEQ Consumption on Benchmark Asset Points (Constant Effort, Worker)

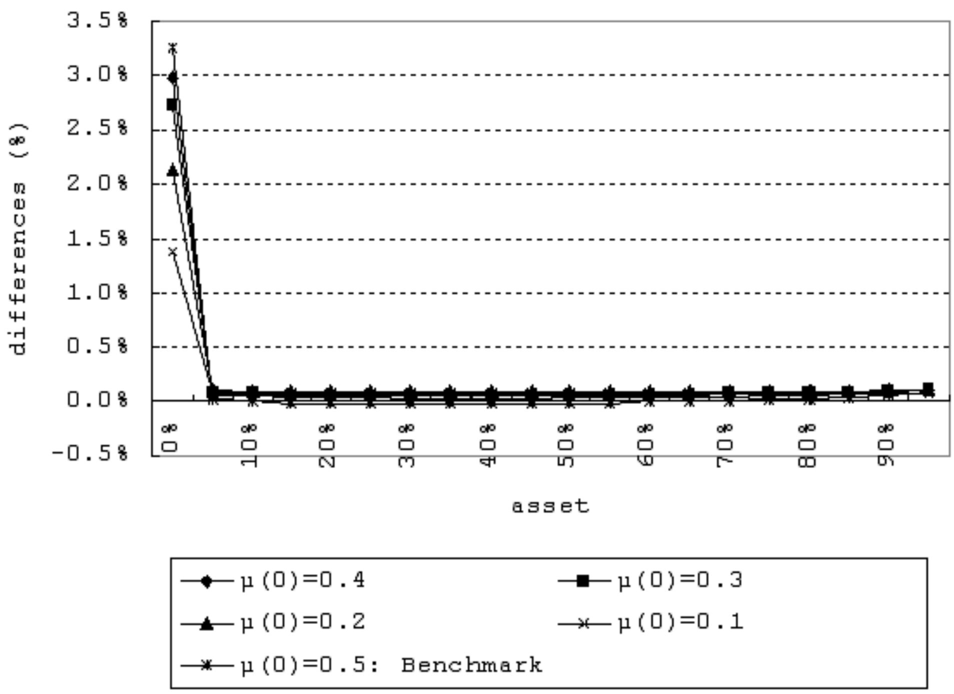

Figure 6: CEQ Consumption on Benchmark Asset Points (Constant Effort, Searcher)

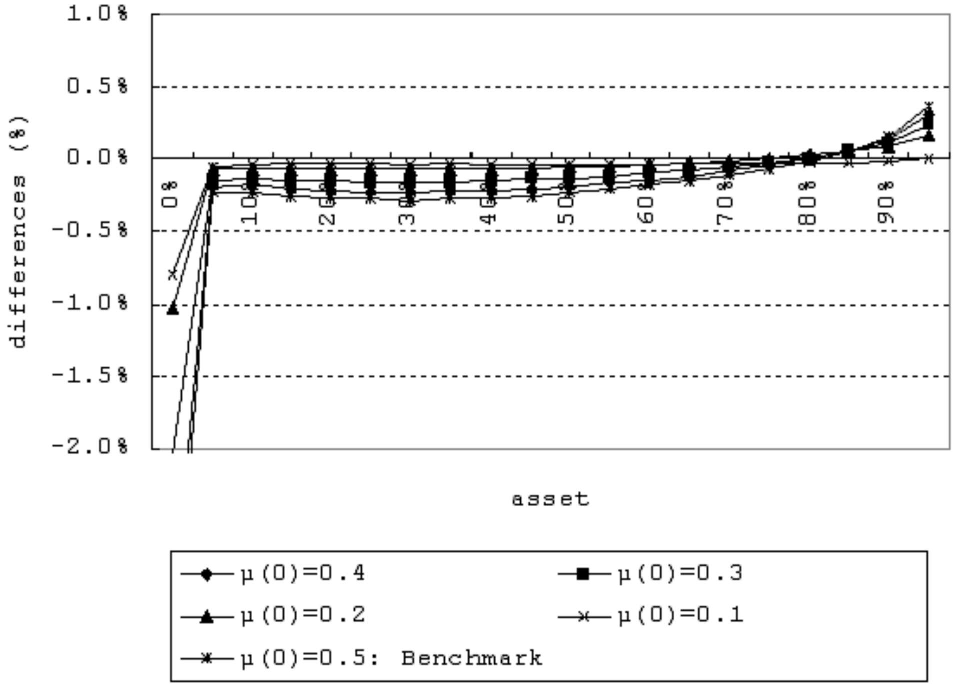

Figure 7: CEQ Consumption for Each Class of Asset (Constant Effort, Worker)

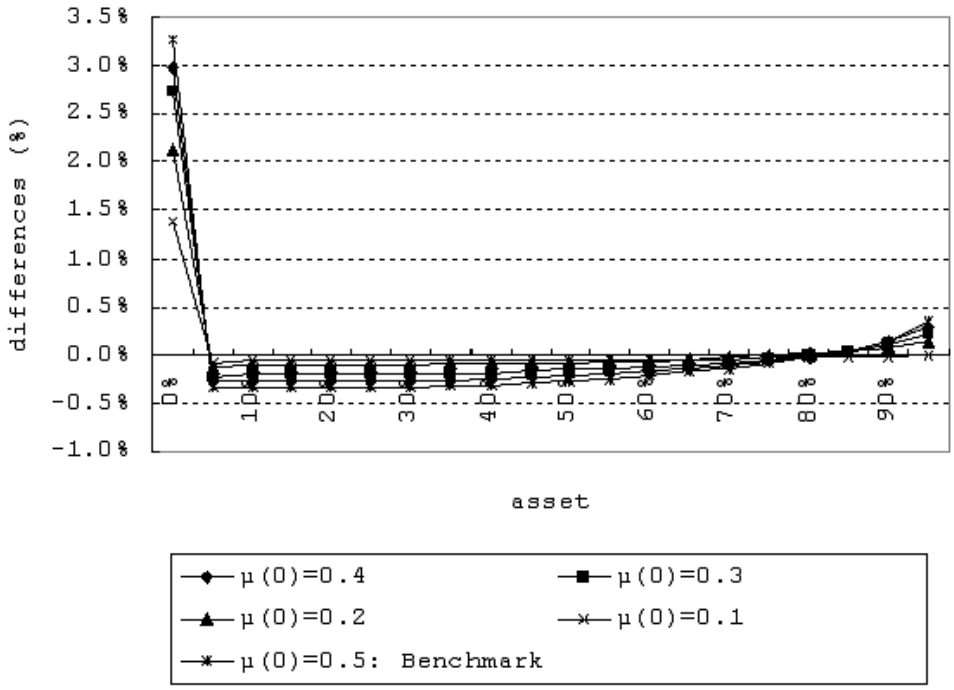

Figure 8: CEQ Consumption for Each Class of Asset (Constant Effort, Searcher)